Three-Phase Circuits

objectives

After studying this unit, you should be able to:

- Discuss the differences between three-phase and single-phase voltages.
- Discuss the characteristics of delta and wye connections.
- Compute voltage and current values for delta and wye circuits.
- Connect delta and wye circuits and make measurements with measuring instruments.

Most of the electrical power generated in the world today is three-phase. Three-phase power was first conceived by Nikola Tesla. In the early days of electric power generation, Tesla not only led the battle concerning whether the nation should be powered with low-voltage direct current or high-voltage alternating current, but he also proved that three-phase power was the most efficient way that electricity could be produced, transmitted, and consumed.

Three-Phase Circuits

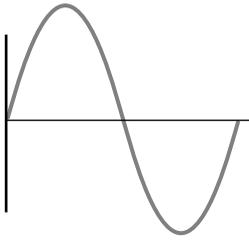There are several reasons why three-phase power is superior to single-phase power.

Figure 12-1 Single-phase power falls to zero three times each cycle.

1. The horsepower rating of three-phase motors and the KVA (kilo-volt-amp) rating of three-phase transformers is about 150% greater than for single-phase motors or transformers with a similar frame size.

2. The power delivered by a single-phase system pulsates, *Figure 12-1*. The power falls to zero three times during each cycle. The power delivered by a three-phase circuit pulsates also, but it never falls to zero, *Figure 12-2*. In a three-phase system, the power delivered to the load is the same at any instant. This produces superior operating characteristics for three-phase motors.
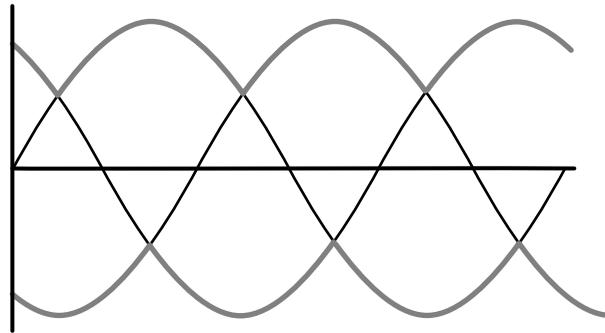

Figure 12-2 Three-phase power never falls to zero.

3. In a balanced three-phase system, the conductors need be only about 75% the size of conductors for a single-phase two-wire system of the same KVA rating. This helps offset the cost of supplying the third conductor required by three-phase systems.

A single-phase alternating voltage can be produced by rotating a magnetic field through the conductors of a stationary coil, as shown in *Figure 12-3*.
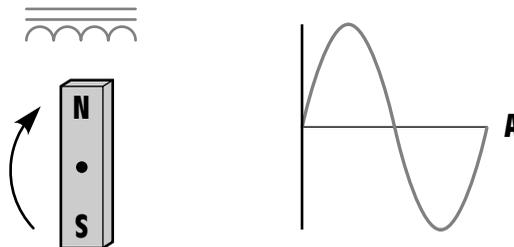

Figure 12-3 Producing a single-phase voltage.

Since alternate polarities of the magnetic field cut through the conductors of the stationary coil, the induced voltage will change polarity at the same speed as the rotation of the magnetic field. The alternator shown in Figure 12-3 is single phase because it produces only one AC voltage.
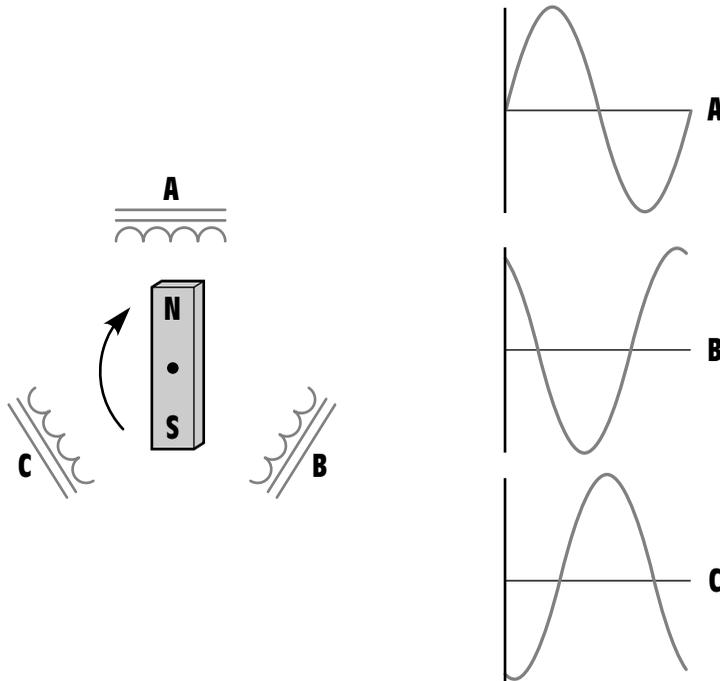

Figure 12-4 The voltages of a three-phase system are 120° out of phase with each other.

Figure 12-5 A wye connection is formed by joining one end of each of the windings together.

If three separate coils are spaced 120° apart, as shown in *Figure 12-4,* three voltages 120° out of phase with each other will be produced when the magnetic field cuts through the coils. This is the manner in which a three-phase voltage is produced. There are two basic three-phase connections, the wye or star connection and the delta connection.

WYE CONNECTION

The **wye** or **star connection** is made by connecting one end of each of the three-phase windings together as shown in *Figure 12-5*. The voltage measured across a single winding or phase is known as the **phase voltage,** as shown in *Figure 12-6*. The voltage measured between the lines is known as the line-to-line voltage or simply as the **line voltage.**
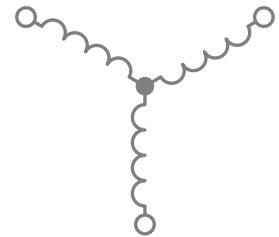
wye connection

star connection
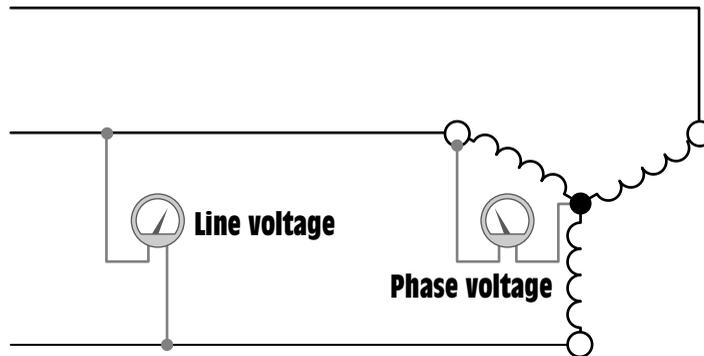
phase voltage

line voltage

Figure 12-6 Line and phase voltages are different in a wye connection.

In *Figure 12-7*, ammeters have been placed in the phase winding of a wye-connected load and in the line supplying power to the load. Voltmeters have been connected across the input to the load and across the phase. A line voltage of 208 V has been applied to the load. Notice that the voltmeter connected across the lines indicates a value of 208 V, but the voltmeter connected across the phase indicates a value of 120 V.

In a wye connected system, the line voltage is higher than the phase voltage by a factor of the square root of 3 (1.732). Two formulas used to compute the voltage in a wye connected system are:

$$E_{Line} = E_{Phase} \times 1.732$$

and

$$E_{Phase} = \frac{E_{Line}}{1.732}$$

> **In a wye-connected system, the line voltage is higher than the phase voltage by a factor of the square root of 3 (1.732).**
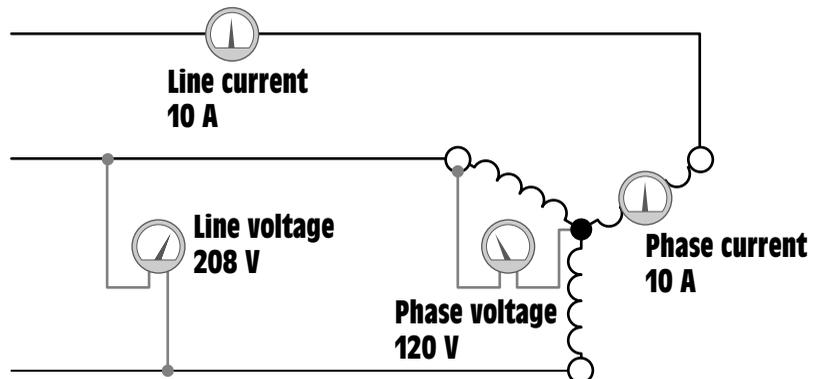

Figure 12-7 Line current and phase current are the same in a wye connection.

Notice in Figure 12-7 that 10 A of current flow in both the phase and the line. **In a wye-connected system, phase current and line current are the same.**

$$I_{Line} = I_{Phase}$$

VOLTAGE RELATIONSHIPS IN A WYE CONNECTION

Many students of electricity have difficulty at first understanding why the line voltage of the wye connection used in this illustration is 208 V instead of 240 V. Since line voltage is measured across two phases that have a value of 120 V each, it would appear that the sum of the two voltages should be 240 V. One cause of this misconception is that many students are familiar with the 240/120 V connection supplied to most homes. If voltage is measured across the two incoming lines, a voltage of 240 V will be seen. If voltage is measured from either of the two lines to the neutral, a voltage of 120 V will be seen. The reason for this is that this is a single-phase connection derived from the center tap of a transformer, *Figure 12-8*. If the center tap is used as a common point, the two line voltages on either side of it will be 180° apart and opposite in polarity, *Figure 12-9*. The vector sum of these two voltages would be 240 V.

Three-phase voltages are 120° apart, not 180°. If the three voltages are drawn 120° apart, it will be seen that the vector sum of these voltages is 208 V,
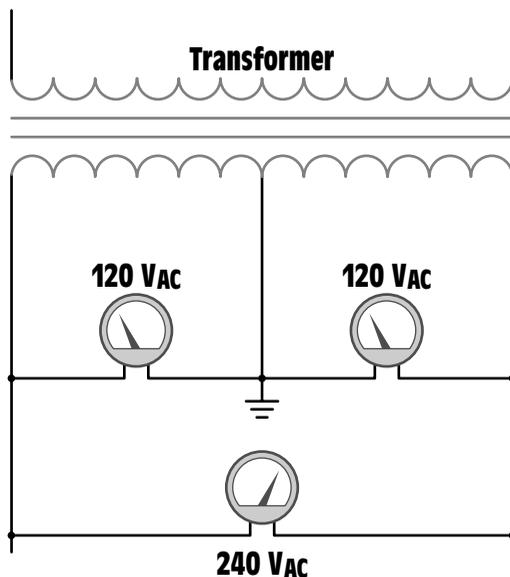
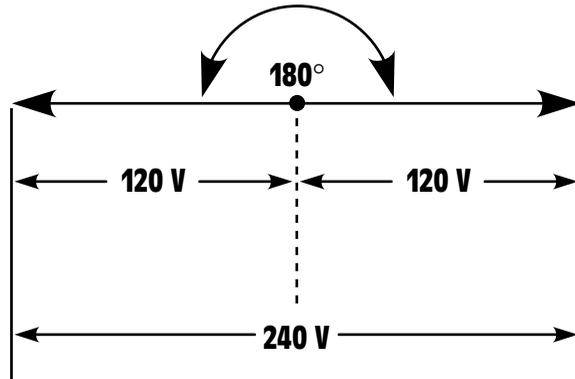**Figure 12-8** Single-phase transformer with grounded center tap.

Figure 12-9 The voltages of a single-phase system are 180° out of phase with each other.
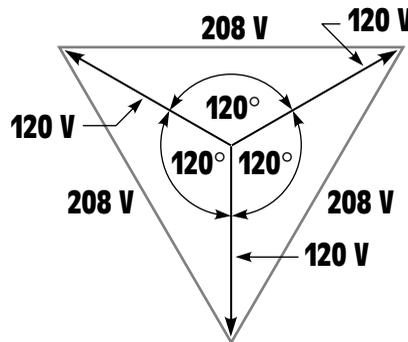

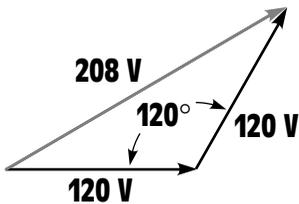**Figure 12-10** Vector sum of the voltages in a three-phase wye connection.

Figure 12-11 Adding voltage vectors of two-phase voltage values.

Figure 12-10. Another illustration of vector addition is shown in *Figure 12-11.* In this illustration two-phase voltage vectors are added and the resultant is drawn from the starting point of one vector to the end point of the other. The parallelogram method of vector addition for the voltages in a wye-connected three-phase system is shown in *Figure 12-12.*

DELTA CONNECTIONS

In *Figure 12-13,* three separate inductive loads have been connected to form a **delta connection.** This connection receives its name from the fact that a schematic diagram of this connection resembles the Greek letter delta (Δ). In *Figure 12-14,* voltmeters have been connected across the lines and
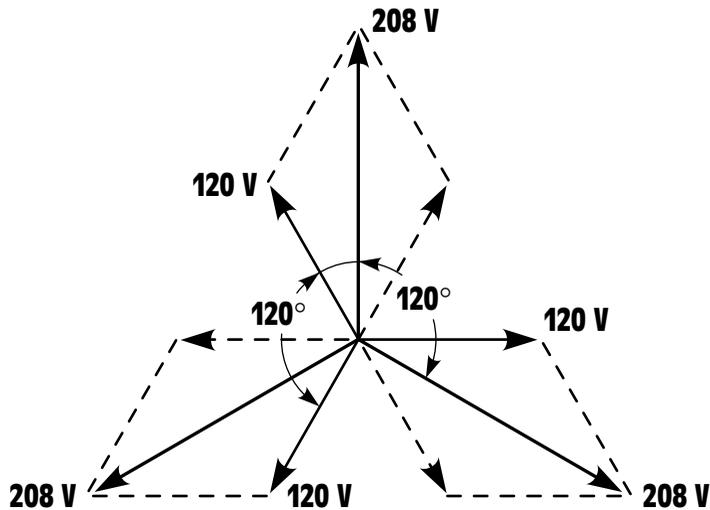
delta connection

Figure 12-12 The parallelogram method of adding three-phase vectors.

Figure 12-13 Three-phase delta connection.

across the phase. Ammeters have been connected in the line and in the phase. **In the delta connection, line voltage and phase voltage are the same.** Notice that both voltmeters indicate a value of 480 V.

$$E_{Line} = E_{Phase}$$

Notice that the line current and phase current are different, however. **The line current of a delta connection is higher than the phase current by a factor of the square root of 3 (1.732).** In the example shown,

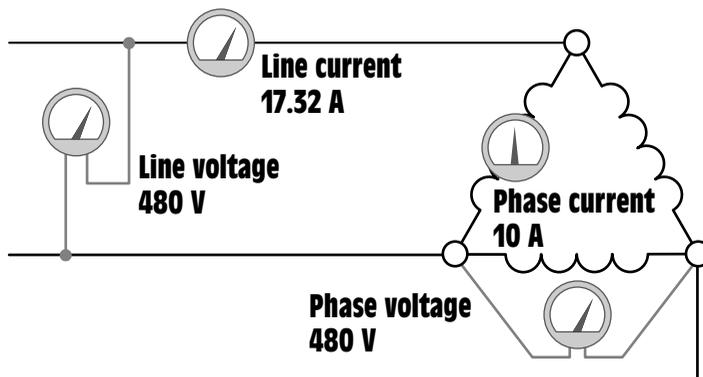**In a delta connection, line voltage and phase voltage are the same.**

The line current of a delta connection is higher than the phase current by a factor of the square root of 3 (1.732).

Figure 12-14 Voltage and current relationships in a delta connection.

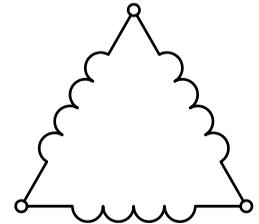it is assumed that each of the phase windings has a current flow of 10 A. The current in each of the lines, however, is 17.32 A. The reason for this difference in current is that current flows through different windings at different times in a three-phase circuit. During some periods of time, current will flow between two lines only. At other times, current will flow from two lines to the third, *Figure 12-15*. The delta connection is similar to a parallel connection because there is always more than one path for current flow. Since these currents are 120° out of phase with each other, vector addition must be used when finding the sum of the currents, *Figure 12-16*.
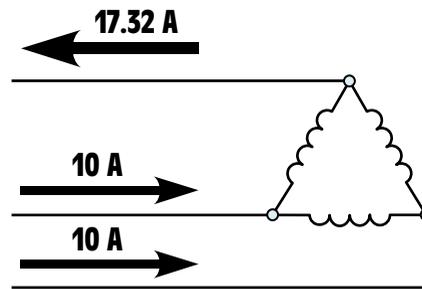
Figure 12-15 Division of currents in a delta connection.
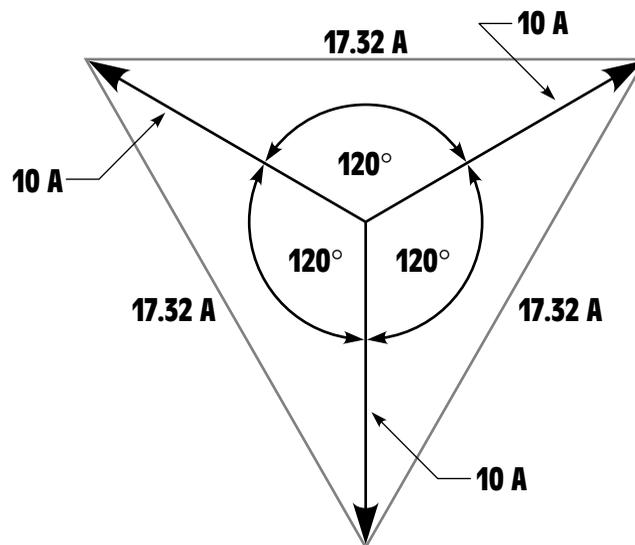
Figure 12-16 Vector addition is used to compute the sum of the currents in a delta connection.

Formulas for determining the current in a delta connection are:

$$I_{Line} = I_{Phase} \times 1.732$$

and

$$I_{Phase} = \frac{I_{Line}}{1.732}$$

THREE-PHASE POWER

Students sometimes become confused when computing power in three-phase circuits. One reason for this confusion is that there are actually two formulas that can be used. If *line* values of voltage and current are known, the power (watts) of a pure resistive load can be computed using the formula:

$$VA = \sqrt{3} \times E_{Line} \times I_{Line}$$

If the *phase* values of voltage and current are known, the apparent power can be computed using the formula:

$$VA = 3 \times E_{Phase} \times I_{Phase}$$

Notice that in the first formula, the line values of voltage and current are multiplied by the square root of 3. In the second formula, the phase values of voltage and current are multiplied by 3. The first formula is used more often because it is generally more convenient to obtain line values of voltage and current, which can be measured with a voltmeter and clamp-on ammeter.

THREE-PHASE CIRCUIT CALCULATIONS

In the following examples, values of line and phase voltage, line and phase current, and power will be computed for different types of three-phase connections.

A wye-connected three-phase alternator supplies power to a delta-connected resistive load, *Figure 12-17*. The alternator has a line voltage of 480 V. Each resistor of the delta load has 8 Ω of resistance. Find the following values:

Example 1

$E_{L(Load)}$ — line voltage of the load

$E_{P(Load)}$ — phase voltage of the load

$I_{P(Load)}$ — phase current of the load

Alternator

E_P
I_P

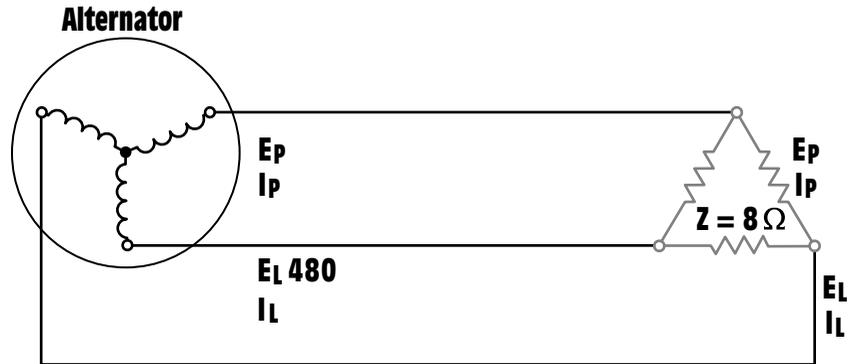E_P
I_P

$Z = 8\,\Omega$

$E_L\ 480$
I_L

E_L
I_L

Figure 12-17 Computing three-phase values using a wye-connected power source and a delta-connected load (Example 1 circuit).

$I_{L(Load)}$ — line current to the load

$I_{L(Alt)}$ — line current delivered by the alternator

$I_{P(Alt)}$ — phase current of the alternator

$E_{P(Alt)}$ — phase voltage of the alternator

P — true power

Solution:

The load is connected directly to the alternator. Therefore, the line voltage supplied by the alternator is the line voltage of the load.

$$E_{L(Load)} = 480\text{ V}$$

The three resistors of the load are connected in a delta connection. In a delta connection, the phase voltage is the same as the line voltage.

$$E_{P(Load)} = E_{L(Load)}$$

$$E_{P(Load)} = 480\text{ V}$$

Each of the three resistors in the load is one phase of the load. Now that the phase voltage is known (480 V), the amount of phase current can be computed using Ohm's Law.

$$I_{P(Load)} = \frac{E_{P(Load)}}{Z}$$

$$I_{P(Load)} = \frac{480}{8}$$

$$I_{P(Load)} = 60\ A$$

The three load resistors are connected as a delta with 60 A of current flow in each phase. The line current supplying a delta connection must be 1.732 times greater than the phase current.

$$I_{L(Load)} = I_{P(Load)} \times 1.732$$

$$I_{L(Load)} = 60 \times 1.732$$

$$I_{L(Load)} = 103.92\ A$$

The alternator must supply the line current to the load or loads to which it is connected. In this example, only one load is connected to the alternator. Therefore, the line current of the load will be the same as the line current of the alternator.

$$I_{L(Alt)} = 103.92\ A$$

The phase windings of the alternator are connected in a wye connection. In a wye connection, the phase current and line current are equal. The phase current of the alternator will, therefore, be the same as the alternator line current.

$$I_{P(Alt)} = 103.92\ A$$

The phase voltage of a wye connection is less than the line voltage by a factor of the square root of 3. The phase voltage of the alternator will be:

$$E_{P(Alt)} = \frac{E_{L(Alt)}}{1.732}$$

$$E_{P(Alt)} = \frac{480}{1.732}$$

$$E_{P(Alt)} = 277.13\ V$$

In this circuit, the load is pure resistive. The voltage and current are in phase with each other, which produces a unity power factor of 1. The true power in this circuit will be computed using the formula:

$$P = 1.732 \times E_{L(Alt)} \times I_{L(Alt)} \times PF$$
$$P = 1.732 \times 480 \times 103.92 \times 1$$
$$P = 86,394.93 \text{ W}$$

Example 2

A delta-connected alternator is connected to a wye-connected resistive load, *Figure 12-18*. The alternator produces a line voltage of 240 V and the resistors have a value of 6 Ω each. The following values will be found:
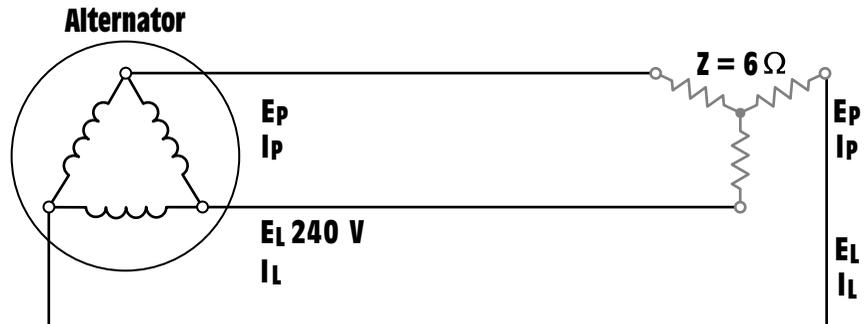

Figure 12-18 Computing three-phase values using a delta-connected source and a wye-connected load (Example 2 circuit).

$E_{L(Load)}$ — line voltage of the load

$E_{P(Load)}$ — phase voltage of the load

$I_{P(Load)}$ — phase current of the load

$I_{L(Load)}$ — line current to the load

$I_{L(Alt)}$ — line current delivered by the alternator

$I_{P(Alt)}$ — phase current of the alternator

$E_{P(Alt)}$ — phase voltage of the alternator

P — true power

Solution:

As was the case in Example 1, the load is connected directly to the output of the alternator. The line voltage of the load must, therefore, be the same as the line voltage of the alternator.

$$E_{L(Load)} = 240 \text{ V}$$

The phase voltage of a wye connection is less than the line voltage by a factor of 1.732.

$$E_{P(Load)} = \frac{240}{1.732}$$

$$E_{P(Load)} = 138.57 \text{ V}$$

Each of the three 6-Ω resistors is one phase of the wye-connected load. Since the phase voltage is 138.57 V, this voltage is applied to each of the three resistors. The amount of phase current can now be determined using Ohm's Law.

$$I_{P(Load)} = \frac{E_{P(Load)}}{Z}$$

$$I_{P(Load)} = \frac{138.57}{6}$$

$$I_{P(Load)} = 23.1 \text{ A}$$

The amount of line current needed to supply a wye-connected load is the same as the phase current of the load.

$$I_{L(Load)} = 23.1 \text{ A}$$

Only one load is connected to the alternator. The line current supplied to the load is the same as the line current of the alternator.

$$I_{L(Alt)} = 23.1 \text{ A}$$

The phase windings of the alternator are connected in delta. In a delta connection the phase current is less than the line current by a factor of 1.732.

$$I_{P(Alt)} = \frac{I_{L(Alt)}}{1.732}$$

$$I_{P(Alt)} = \frac{23.1}{1.732}$$

$$I_{P(Alt)} = 13.34 \text{ A}$$

The phase voltage of a delta is the same as the line voltage.

$$E_{P(Alt)} = 240 \text{ V}$$

Since the load in this example is pure resistive, the power factor has a value of unity, or 1. Power will be computed by using the line values of

voltage and current.

$$P = 1.732 \times E_L \times I_L \times PF$$
$$P = 1.732 \times 240 \times 23.1 \times 1$$
$$P = 9{,}602.21 \text{ W}$$

Example 3

The phase windings of an alternator are connected in wye. The alternator produces a line voltage of 440 V, and supplies power to two resistive loads. One load contains resistors with a value of 4 Ω each, connected in wye. The second load contains resistors with a value of 6 Ω each, connected in delta, *Figure 12-19*. The following circuit values will be found.
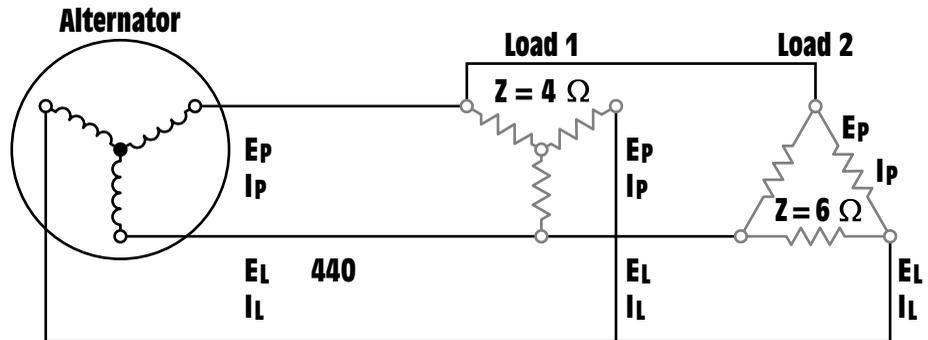

Figure 12-19 Computing three-phase values using a wye-connected source and two three-phase loads (Example 3 circuit).

$E_{L(Load\ 2)}$ — line voltage of load 2

$E_{P(Load\ 2)}$ — phase voltage of load 2

$I_{P(Load\ 2)}$ — phase current of load 2

$I_{L(Load\ 2)}$ — line current to load 2

$E_{P(Load\ 1)}$ — phase voltage of load 1

$I_{P(Load\ 1)}$ — phase current of load 1

$I_{L(Load\ 1)}$ — line current to load 1

$I_{L(Alt)}$ — line current delivered by the alternator

$I_{P(Alt)}$ — phase current of the alternator

$E_{P(Alt)}$ — phase voltage of the alternator

P — true power

Solution:

Both loads are connected directly to the output of the alternator. The line voltage for both loads 1 and 2 will be the same as the line voltage of the alternator.

$$E_{L(Load\ 2)} = 440\ V$$

$$E_{L(Load\ 1)} = 440\ V$$

Load 2 is connected as a delta. The phase voltage will be the same as the line voltage.

$$E_{P(Load\ 2)} = 440\ V$$

Each of the resistors that constitutes a phase of load 2 has a value of 6 Ω. The amount of phase current can be found using Ohm's Law.

$$I_{P(Load\ 2)} = \frac{E_{P(Load\ 2)}}{Z}$$

$$I_{P(Load\ 2)} = \frac{440}{6}$$

$$I_{P(Load\ 2)} = 73.33\ A$$

The line current supplying a delta-connected load is 1.732 times greater than the phase current. The amount of line current needed for load 2 can be computed by increasing the phase current value by 1.732.

$$I_{L(Load\ 2)} = I_{P(Load\ 2)} \times 1.732$$

$$I_{L(Load\ 2)} = 73.33 \times 1.732$$

$$I_{L(Load\ 2)} = 127.01\ A$$

The resistors of load 1 are connected to form a wye. The phase voltage of a wye connection is less than the line voltage by a factor of 1.732.

$$E_{P(Load\ 1)} = \frac{E_{L(Load\ 1)}}{1.732}$$

$$E_{P(Load\ 1)} = \frac{440}{1.732}$$

$$E_{P(Load\ 1)} = 254.04\ V$$

Now that the voltage applied to each of the 4 Ω resistors is known, the phase current can be computed using Ohm's Law.

$$I_{P(Load\ 1)} = \frac{E_{P(Load\ 1)}}{Z}$$

$$I_{P(Load\ 1)} = \frac{254.04}{4}$$

$$I_{P(Load\ 1)} = 63.51\ A$$

The line current supplying a wye-connected load is the same as the phase current. Therefore, the amount of line current needed to supply load 1 is:

$$I_{L(Load\ 1)} = 63.51\ A$$

The alternator must supply the line current needed to operate both loads. In this example, both loads are resistive. The total line current supplied by the alternator will be the sum of the line currents of the two loads.

$$I_{L(Alt)} = I_{L(Load\ 1)} + I_{L(Load\ 2)}$$

$$I_{L(Alt)} = 63.51 + 127.01$$

$$I_{L(Alt)} = 190.52\ A$$

Since the phase windings of the alternator in this example are connected in a wye, the phase current will be the same as the line current.

$$I_{P(Alt)} = 190.52\ A$$

The phase voltage of the alternator will be less than the line voltage by a factor of 1.732.

$$E_{P(Alt)} = \frac{440}{1.732}$$

$$E_{P(Alt)} = 254.04\ V$$

Both of the loads in this example are resistive and have a unity power factor of 1. The total power in this circuit can be found by using the line voltage and total line current supplied by the alternator.

$$P = 1.732 \times E_L \times I_L \times PF$$

$$P = 1.732 \times 440 \times 190.52 \times 1$$

$$P = 145,191.48\ W$$

SUMMARY

1. The voltages of a three-phase system are 120° out of phase with each other.

2. The two types of three-phase connections are wye and delta.

3. Wye connections are characterized by the fact that one terminal of each device is connected together.

4. In a wye connection, the phase voltage is less than the line voltage by a factor of 1.732. The phase current and line current are the same.

5. In a delta connection, the phase voltage is the same as the line voltage. The phase current is less than the line current by a factor of 1.732.

REVIEW QUESTIONS

1. How many degrees out of phase with each other are the voltages of a three-phase system?

2. What are the two main types of three-phase connections?

3. A wye-connected load has a voltage of 480 V applied to it. What is the voltage drop across each phase?

4. A wye-connected load has a phase current of 25 A. How much current is flowing through the lines supplying the load?

5. A delta connection has a voltage of 560 V connected to it. How much voltage is dropped across each phase?

6. A delta connection has 30 A of current flowing through each phase winding. How much current is flowing through each of the lines supplying power to the load?

7. A three-phase resistive load has a phase voltage of 240 V and a phase current of 18 A. What is the power of this load?

8. If the load in question 7 is connected in a wye, what would be the line voltage and line current supplying the load?

9. An alternator with a line voltage of 2400 V supplies a delta-connected load. The line current supplied to the load is 40 A. Assuming the load is a balanced three-phase load, what is the impedance of each phase?

10. If the load is pure resistive, what is the power of the circuit in question 9?

1. Refer to the circuit shown in Figure 12-17 to answer the following questions. It is assumed the alternator has a line voltage of 240 V and the load has an impedance of 12 Ω per phase. Find all missing values.

$E_{P(A)}$ _____ $E_{P(L)}$ _____

$I_{P(A)}$ _____ $I_{P(L)}$ _____

$E_{L(A)}$ 240 $E_{L(L)}$ _____

$I_{L(A)}$ _____ $I_{L(L)}$ _____

P _____ $Z_{(PHASE)}$ 12 Ω

2. Refer to the circuit shown in Figure 12-18 to answer the following questions. Assume the alternator has a line voltage of 4,160 V and the load has an impedance of 60 Ω per phase. Find all missing values.

$E_{P(A)}$ _____ $E_{P(L)}$ _____

$I_{P(A)}$ _____ $I_{P(L)}$ _____

$E_{L(A)}$ 4160 V $E_{L(L)}$ _____

$I_{L(A)}$ _____ $I_{L(L)}$ _____

P _____ $Z_{(PHASE)}$ 60 Ω

3. Refer to the circuit shown in Figure 12-19 to answer the following questions. It is assumed that the alternator has a line voltage of 560 V. Load 1 has an impedance of 5 Ω per phase and load 2 has an impedance of 8 Ω per phase. Find all missing values.

$E_{P(A)}$ _____ $E_{P(L1)}$ _____ $E_{P(L2)}$ _____

$I_{P(A)}$ _____ $I_{P(L1)}$ _____ $I_{P(L2)}$ _____

$E_{L(A)}$ 560 V $E_{L(L1)}$ _____ $E_{L(L2)}$ _____

$I_{L(A)}$ _____ $I_{L(L1)}$ _____ $I_{L(L2)}$ _____

P _____ $Z_{(PHASE)}$ 5 Ω $Z_{(PHASE)}$ 8 Ω