Center Tapped Transformer and 120/240 Volt Secondary Models

W. H. Kersting, *Life Fellow, IEEE*
Milsoft Utility Solutions

Abstract – ***Distribution engineers have treated the standard "single phase" distribution transformer connection as single phase because from the primary side of the transformer these connections are single phase and in the case of standard rural distribution single phase line to ground. However, with the advent of detailed circuit modeling we are beginning to see distribution modeling and analysis being accomplished past the transformer to the secondary. Which now brings into focus the reality that standard 120/240 secondary systems are not single phase line to ground systems, instead they are three wire systems with two phases and one ground wires. Further, the standard 120/240 secondary is different from the two phase primary system in that the secondary phases are separated by 180 degrees instead of three phases separated by 120 degrees. What all of this means is that analysis software and methods must now deal with an electrical system requiring a different set of algorithms than those used to model and analyze the primary system.***

This paper will describe the modeling and analysis of the single-phase center tap transformer serving 120 Volt and 240 Volt single-phase loads from a three-wire secondary.

Index Terms – ***Distribution systems, modeling, analysis, transformers, secondary***

I. INTRODUCTION

The analysis of three-phase unbalanced distribution feeders normally models the load of distribution transformers at the primary terminals of the transformer. For single-phase center tapped transformers serving a three-wire secondary the load will serve two120 volt loads and a 240 volt load. The actual load on the transformer will be the sum of these loads. Unfortunately, the actual customer loading is generally not known so that some form of load allocation will be necessary to model the load of the transformer for analysis purposes. [1] This works reasonably well when power flow studies are performed on the primary feeder system. The question is will the computed voltage be sufficient to make sure that every customer's voltage will satisfy the ANSI standard? [2] Many times "rules of thumb" are used to determine the minimum acceptable transformer voltage. One such rule assumes a 1 volt drop on the service drop, two volts on the secondary and three volts on the transformer.

W. H. Kersting is a consultant to Milsoft Utility Solutions and a partner in WH Power Consultants, Las Cruces, NM.

Corresponding e-mail for this paper: bjkersting@zianet.com

With those assumptions the minimum acceptable transformer primary voltage is 120 volts. Is that good enough? What is needed is a method to model and analyze the centered tapped transformer and the secondary system once the primary voltage on the transformer has been determined from a feeder power-flow study. This paper will develop a method to accurately model the center tapped transformer and the three wire secondary system.

II. The Center Tapped Transformer Model

The model of a center tapped single-phase transformer connected line-to-ground serving two 120 volt loads and one 240 volt load through a triplex cable is shown in Figure 1.

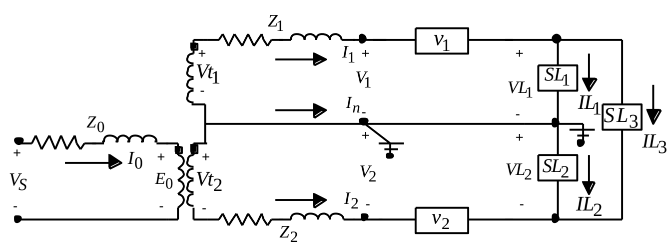

Figure 1 – Center Tapped Single-Phase Transformer

Typically the total impedance of the transformer ($R_T + jX_T$) is known. This is usually expressed in per unit on the rating of the transformer as the bases. Figure 1 shows that the known transformer impedance needs to be broken into three parts. For interlaced design the three impedances are given by [3]:

$$\begin{aligned} Z_0 &= 0.5 \cdot R_T + j0.8 \cdot X_T \\ Z_1 &= R_T + j0.4 \cdot X_T \qquad \text{per unit} \\ Z_2 &= R_T + j0.4 \cdot X_T \end{aligned} \tag{1}$$

In Equations 1 the impedances must be converted to ohms relative to the respective sides of the transformer.

In Figure 1 define:

$$n_t = \frac{\text{High Side Rated Voltage}}{\text{Low Side-Half Winding Rated Voltage}} \tag{2}$$

Example: $n_t = \frac{7200}{120} = 60$ (3)

In this example when converting from per unit to ohms, the base voltage on the high side is 7200 volts and the base voltage on the low side is 120 volts.

A modified forward/backward sweep iterative technique is used to analyze the center tapped transformer. In this technique the forward sweep is used to compute the node voltages downstream using the present value of the line currents. The backward sweep is used to calculate the new load and line currents using the most recent node voltages. To start the process all line currents are assumed to be zero so that during the first pass all node voltages will be the nominal values. The forward/backward sweeps continue until the difference between the present and previous node voltages at all nodes is less than a specified tolerance. Once convergence has been achieved the primary line current is computed using Equation 4. [4]

$$[I_{00}] = [d_t] \cdot [I_{12}] \tag{4}$$

where:

$$[I_{00}] = \begin{bmatrix} I_0 \\ I_0 \end{bmatrix} \qquad [I_{12}] = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{5}$$

and:

$$[d_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \tag{6}$$

The forward sweep equation is given by Equation 7. [4]

$$[V_{12}] = [A_t] \cdot [V_{ss}] - [B_t] \cdot [I_{12}] \tag{7}$$

where:

$$[A_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad [B_t] = \begin{bmatrix} Z_1 + \frac{1}{n_t^2} \cdot Z_0 & -\frac{1}{n_t^2} \cdot Z_0 \\ \frac{1}{n_t^2} \cdot Z_0 & -\left(Z_2 + \frac{1}{n_t^2} \cdot Z_0 \right) \end{bmatrix} \tag{8}$$

III. The Triplex Cable Model

The triplex cable consists of two insulated conductors and one bare conductor as shown in Figure 2.

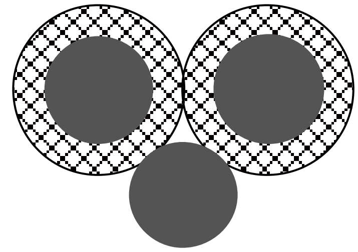

Figure 2 – Triplex Cable

The triplex cable can be modeled with 2x2 matrices [4]. With reference to Figure 1, the forward sweep matrix equation for the triplex cable is:

$$[VL_{12}] = [A_s] \cdot [V_{12}] - [B_s] \cdot [I_{12}] \tag{9}$$

The triplex impedances are computed by applying Carson's Equations to compute the 3x3 primitive impedance matrix. The primitive impedance matrix in partitioned form is: [4]

$$[zp] = \begin{bmatrix} [zp_{ij}] & [zp_{in}] \\ [zp_{nj}] & [zp_{nn}] \end{bmatrix} \tag{10}$$

The partitioned primitive impedance matrix is reduced to the 2x2 phase impedance matrix by applying the Kron reduction [4].

$$[Z_s] = \left([zp_{ij}] - [zp_{in}] \cdot [zp_{nn}]^{-1} \cdot [zp_{nj}] \right) \cdot length \tag{11}$$

In the process of applying the Kron reduction, the neutral current transformer matrix is determined which allows the computation of the current flowing in the neutral wire to be computed once the line currents have been determined.

$$I_n = [tn] \cdot [I_{12}] \qquad \text{where: } [tn] = -[zp_{nn}]^{-1} \cdot [zp_{nj}] \tag{12}$$

Once the secondary phase impedance matrix has been computed two KVL loop equations can be written to compute the voltage drop down the secondary. Referring to Figure 1 the two equations are:

$$\begin{aligned} VL_1 &= V_1 - v_1 = V_1 - (Zs_{11} \cdot I_1 + Zs_{12} \cdot I_2) \\ VL_2 &= V_2 + v_2 = V_2 + (Zs_{21} \cdot I_1 + Zs_{22} \cdot I_2) \end{aligned} \tag{13}$$

In matrix form Equation becomes:

$$\begin{gathered} \begin{bmatrix} VL_1 \\ VL \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} - \begin{bmatrix} Zs_{11} & Zs_{12} \\ -Zs_{21} & -Zs_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \\ [VL_{12}] = [A_s] \cdot [V_{12}] - [B_s] \cdot [I_{12}] \end{gathered} \tag{14}$$

Equation 14 is used for the forward sweep. In the backward sweep for this simple system the input and output currents are the same so no equations are needed.

IV. The Single-Phase Loads

In Figure 1 the single-phase loads are given by:

$$\begin{aligned} S_1 &= P_1 + jQ_1 \\ S_2 &= P_2 + jQ_2 \quad \text{kW +jkvar} \\ S_3 &= P_3 + jQ_3 \end{aligned} \tag{15}$$

The single-phase load currents are computed by:

$$IL_n = \left(\frac{SL_n}{VL_n}\right)^* \tag{16}$$

The line current vector $[I_{12}]$ is computed using Equation 17.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} IL_1 \\ IL_2 \\ IL_3 \end{bmatrix}$$

$$[I_{12}] = [CL] \cdot [IL_{123}] \tag{17}$$

When the system has converged the current flowing in the neutral conductor is computed using Equation 12 and then the current flowing in dirt (ground) is determined by:

$$I_g = -(I_1 + I_2 + I_n) \tag{18}$$

V. The Modified Forward/Backward Sweep Technique

To start the iterative technique the line current vector $[I_{12}]$ is set to zero and the primary voltage vector $[V_{ss}]$ is set equal to the specified primary voltage of the transformer. Equation 7 is used to compute the low side transformer terminal voltages and then Equation 14 is used to compute the load voltages. The backward sweep begins by applying Equation 16 to compute the load currents and then Equation 17 is used to compute the line current vector. For this very simple system the forward sweep uses the new line currents to compute the new load voltages. This process continues until the difference between the previous and present load voltages are less than a specified tolerance.

V. Example 1

A 50 kVA, 7200-120/240 voltage center tapped transformer with an impedance of 0.011 + j0.018 per unit serves the following loads through 100 ft. of triplex cable. The triplex cable has two #2/0 AA phase conductors and one bare #2/0 ACSR neutral conductor. The loads served are:

120 V load #1: 5 kVA, 0.95 power factor
120 V. load #2: 10 kVA, 0.90 power factor
240 V. load #3: 25 kVA, 0.85 power factor

The triplex secondary matrices for the forward sweep are computed to be:

$$[A_s] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[B_s] = \begin{bmatrix} 0.0237 + j0.0155 & 0.0091 + j0.0085 \\ -0.0091 - j0.0085 & -0.0237 - j0.0155 \end{bmatrix}$$

The matrices for the center tapped transformer are:

$$[A_t] = \begin{bmatrix} 0.0167 & 0 \\ 0 & 0.0167 \end{bmatrix}$$

$$[B_t] = \begin{bmatrix} 0.0048 + j0.0062 & -0.0016 - j0.0041 \\ 0.0016 + j0.0041 & -0.0048 - j0.0062 \end{bmatrix}$$

Using the modified forward/backward sweep method, the results are:

Transformer secondary voltages:

$$\begin{aligned} V_{1n} &= 118.27\underline{/-0.50} \\ V_{2n} &= 118.10\underline{/-0.51} \end{aligned} \quad \text{Volts}$$

Load voltages:

$$\begin{aligned} VL_{1n} &= 116.67\underline{/-0.16} \\ VL_{2n} &= 114.78\underline{/-0.14} \\ VL_{12} &= 231.45\underline{/-0.15} \end{aligned} \quad \text{Volts}$$

Primary line current:

$$I_0 = 5.74\underline{/-28.76} \quad \text{Amps}$$

The secondary line currents:

$$\begin{aligned} I_1 &= 150.00\underline{/-28.09} \\ I_2 &= 194.88\underline{/150.72} \\ I_{neutral} &= 32.22\underline{/-5.57} \\ I_{dirt} &= 22.24\underline{/-75.48} \end{aligned} \quad \text{Amps}$$

For this simple system the voltage unbalance for the 120 volt loads is demonstrated. Both of these voltages do provide the ANSI [2] minimum service voltage of 114 volts. Also the use of Equations 12 and 18 demonstrate how the currents flowing in neutral and dirt are calculated.

VI. Short Circuit Analysis

The short circuit currents for the center tapped transformer/secondary are of great interest. To calculate these currents the matrices used for the power-flow calculations do not work. Instead, basic circuit analysis equations are developed that will allow for the calculation of all possible short circuit currents. The circuit for a center tapped transformer connected from phase A to Ground is shown in Figure 3. The voltage drop v_A represents the voltage drop on phase A from the Thevenin source to the transformer terminals.

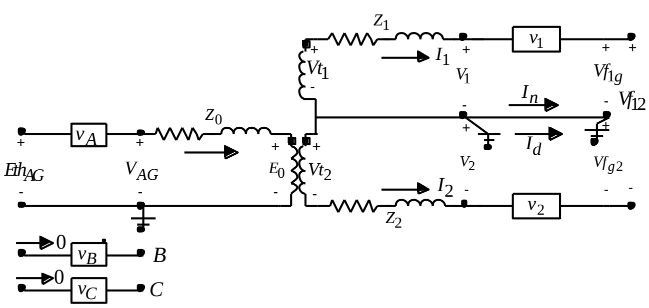

Figure 3 – Short Circuit System

In Figure 3 the voltage Eth_{AG} will typically be the nominal line-to-neutral voltage of the primary system. Since the transformer is connected to phase A, for short circuits in the transformer/secondary system there will not be any currents in phases B and C on the primary. The only voltage drop on the primary will be that on phase A. In general the voltage drops on the primary are given by:

$$\begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ Z_{BA} & Z_{BB} & Z_{BC} \\ Z_{CA} & Z_{CB} & Z_{CC} \end{bmatrix} \cdot \begin{bmatrix} I_A \\ 0 \\ 0 \end{bmatrix} \quad (19)$$

In Equation 19 the impedance matrix will be the equivalent impedance from the transformer terminals back to the equivalent feeder source. Many distribution analysis programs will give the positive and zero sequence impedances at the terminals of the transformer for faults at the terminals of the transformer. When this is the case, the impedance matrix of Equation 19 will be given by:

$$[Z_{ABC}] = [A] \cdot [Z_{012}] \cdot [A]^{-1}$$

$$[Z_{ABC}] = \begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ Z_{BA} & Z_{BB} & Z_{BC} \\ Z_{CA} & Z_{CB} & Z_{CC} \end{bmatrix} \quad (20)$$

$$\text{where: } [A] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad a = 1.0\underline{/120}$$

$$[Z_{012}] = \begin{bmatrix} Z_{zero} & 0 & 0 \\ 0 & Z_{pos} & 0 \\ 0 & 0 & Z_{neg} \end{bmatrix}$$

With the currents in phase B and C zero, the voltage drop on phase A is given by:

$$v_a = Z_{AA} \cdot I_A \quad (21)$$

In Figure 3 there are 17 variables which means that there must be 17 independent equations in order to solve for the various short circuit currents. Fifteen of the equations will be general and used for the four possible types of short circuits. The 15 equations are:

$$\begin{aligned}
&1.\ Eth_{\mathrm{AG}} = V_{AG} + v_A \\
&2.\ 0 = -V_{AG} + E_0 + Z_0 \cdot \mathrm{I_A} \\
&3.\ 0 = -v_A + Z_{AA} \cdot I_A \\
&4.\ 0 = -E_0 + n_t \cdot Vt_1 \\
&5.\ 0 = -E_0 + n_t \cdot Vt_2 \\
&6.\ 0 = -I_A + \frac{1}{n_t} \cdot (I_1 - I_2) \\
&7.\ 0 = -Vt_1 + V_1 + Z_1 \cdot I_1 \\
&8.\ 0 = -Vt_2 + V_2 - Z_2 \cdot I_2 \\
&9.\ 0 = -V_1 + Vf_{1g} + v_1 \\
&10.\ 0 = -V_2 + Vf_{g2} - v_2 \\
&11.\ 0 = -Vf_{12} + Vf_{1g} + Vf_{g2} \\
&12.\ 0 = -v_1 + Zs_{11} \cdot I_1 + Zs_{12} \cdot I_2 \\
&13.\ 0 = -v_2 + Zs_{21} \cdot I_1 + Zs_{22} \cdot I_2 \\
&14.\ 0 = -\mathrm{I_n} + tn_{11} \cdot I_1 + tn_{12} \cdot I_2 \\
&15.\ 0 = -I_d + I_1 + I_2 + I_n
\end{aligned} \quad (22)$$

The final two independent equations are a function of the type of short circuit and are given as:

$$\text{Phase 1 to ground:} \qquad \begin{aligned} Vf_{1g} &= 0 \\ I_2 &= 0 \end{aligned} \quad (23)$$

Phase 2 to ground:
$$\begin{aligned} Vf_{g2} &= 0 \\ I_1 &= 0 \end{aligned} \tag{24}$$

Phase 1 to 2:
$$\begin{aligned} Vf_{12} &= 0 \\ I_n + I_d &= 0 \end{aligned} \tag{25}$$

Phase 1 to 2 to ground:
$$\begin{aligned} Vf_{1g} &= 0 \\ Vf_{g2} &= 0 \end{aligned} \tag{26}$$

For the system of Figure 3, the Thevenin equivalent sequence impedances and source voltage are:

$$\begin{aligned} Z_{zero} &= 3.2671 + j9.5352 \\ Z_{pos} &= Z_{neg} = 0.9298 + j2.9839\ \Omega \\ Eth_{AG} &= 7200\underline{/0} \end{aligned} \tag{27}$$

Applying Equation 20 the self impedance of phase A is:

$$Z_{AA} = 1.7089 + j5.1677 \tag{28}$$

Table 1 gives the maximum short circuit currents for short circuits at the load terminals and the secondary terminals of the transformer.

Table 1

Maximum Short Circuit Currents

	Transformer Terminals		Load Terminals	
	LG	LL&LLG	LG	LL&LLG
I_A	215.7	264.7	57.6	146.7
I_1	12,945	7,941	3,457	4,440
I_n	9,268	0	2,475	0
I_d	6,397	0	1,708	0

Because the transformer secondary impedances (Z_1 & Z_2) are equal, for all short circuit conditions the currents in the two secondary legs will be equal. Also, since the two impedances are equal the line-to-line and the line-to-line-to-ground short circuit currents will be equal.

Table VIII The IEEE 34 Node Test Feeder Power Flow

At the IEEE PES 2008 Transmission and Distribution Conference as part of a panel session a paper[1] was presented that modeled the IEEE 34 Node Test Feeder [5] using Automatic Meter Reading data for the loads on the transformers. The data used were for the 314 customers connected to 23 different single-phase transformers. Data for a three month period in 2006 were used. Because the sample data did not come from transformers connected to an actual feeder, the data was used for modeling the loads in the 34 node test feeder. The maximum diversified demand using the AMR readings of the 314 customers was 1198 kW that occurred in the fifteen minute period between 5:45 and 6:00 p.m. on July 17, 2006. The 15 minute kW demands for the 23 transformers during the peak period are shown in Figure 4.

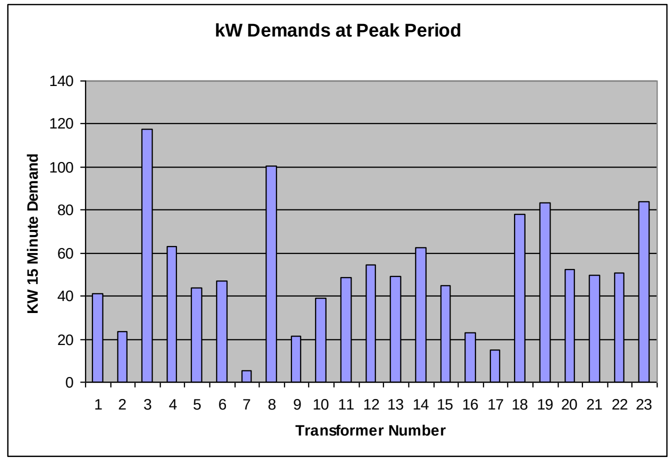

Figure 4 – Transformer kW 15 Minute Demands

The 100 kVA transformer (T-4) connected at node 818 serving 28 customers was of interest. The 15 minute kW demand used as the load in the base case power flow study was 63.15 kW. In this study the voltage at the primary terminals of the transformer was 121.7 volts on a 120 volt base. Using the "rule of thumb" defined earlier this voltage should provide every customer with a voltage within the ANSI standard of 114 volts at the meter. For this case was every customer's voltage at least 114 volts? In order to answer that question a typical secondary for that transformer is modeled as shown in Figure 5

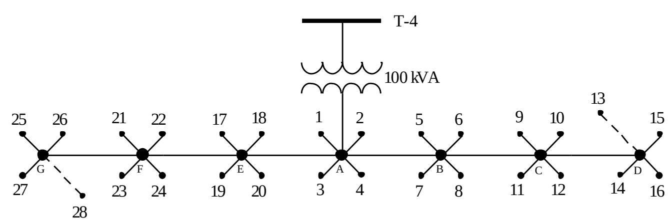

Figure 5 – Secondary System

Figure 5 shows 7 poles serving the 28 customers. The individual customer 15 minute kW demands during the peak period are shown in Figure 6.

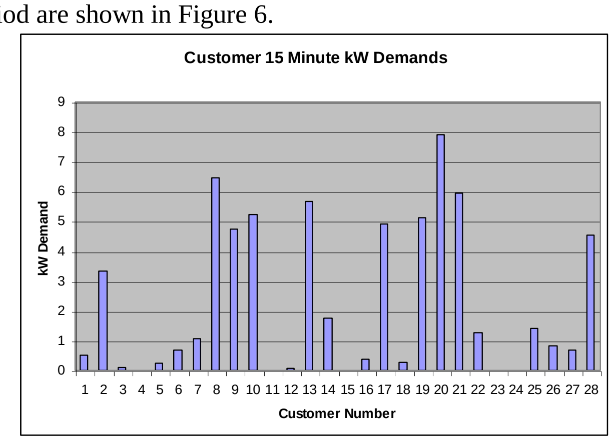

Figure 6 – Customer kW Demands

In Figure 6 the spacing between the poles is 150 ft. The secondary consists of a 2/0 AA triplex cable. Customers #13 and #28 are the most remote customers and those loads are served from 75 ft., of #4 ACSR triplex cable service drop. The transformer is rated 100 kVA, 14,400-120/240 with an impedance of 1.6 + j1.4 %. Load data for the poles and customers 13 and 28 are given in Table 2.

Table 2

kW + jkvar Loads

Node	Load 1	Load 2	Load 3
	kW + jkvar	kW + jkvar	kW + jkvar
A	0.79 + j0.26	1.18 + j0.39	1.97 + j0.95
B	2.54 + j0.83	1.69 + j0.56	4.23 + j1.05
C	2.00 + j0.66	3.01 + j0.99	5.01 + j2.43
D	0.43 + j0.14	0.65 + j0.21	1.08 + j0.52
Cust. #13	1.14 + j0.37	1.71 + j0.56	2.84 + j1.38
E	3.64 + j1.12	5.46 + j1.79	9.10 + j4.41
F	2.16 + j0.71	1.44 + j0.47	3.60 + j1.74
G	1.50 + j0.49	2.25 + j0.74	3.74 + j1.81
Cust. #28	0.59 + j0.19	0.88 + j0.29	1.47 + j0.71

In Table 2, with the exception of nodes D and G, the kW and kvar demands of the four customers connected to the node are summed. The assumption is made that half the total load is the 120 volt loads and the other half of the load is the 240 volt load. The loads on node D and G is the sum of three customer loads. For customers #13 and #28 half of the load is considered the 120 volt loads and the other half is the 240 volt load.

The secondary circuit of Figure 5 with the loads of Table 2 was modeled using the Milsoft Utility Services distribution analysis program Windmil. [7] Table 3 gives the resulting voltages.

Table 3

Node and Customer Voltages

Node	V1	V2	V12
A	120.0	119.9	239.9
B	117.6	117.0	234.6
C	116.2	114.6	230.8
D	115.6	113.6	229.2
Cust. #13	114.8	112.5	227.2
E	116.9	115.6	232.5
F	115.1	113.7	228.8
G	114.4	112.3	226.7
Cust. #28	113.9	111.7	225.6

Table 3 gives the node and customer voltages for the secondary system when the feeder is experiencing the maximum diversified demand for the year. It is noted that the customers at nodes D, F and G will not receive voltages within the ANSI standard. In particular customers #13 and #28 are well below the ANSI standard. When this is the case changes to the secondary system must be made.

The AMR readings show that transformer T-4 has a maximum diversified demand of 63.15 kW during the feeder peak. The data also shows that transformer T-4 maximum diversified demand of 85.37 kW occurs in the time period between 8:30 and 8:45 p.m. With such a difference further studies should be performed for that time period. The feeder diversified demand in that time period is 994.6 kW so there is a chance that the primary voltage of T-4 will be higher in this time period so that the secondary voltages will not be less than those during the feeder's peak period.

VIII. The IEEE 34 Node Test Feeder Short Circuits

For the short circuit studies on the secondary side of the center-tap transformer it is necessary to know the equivalent impedances from the primary of the transformer back to the primary side of the substation transformer. The Thevnin impedance from the primary of the substation transformer back to the equivalent system source is considered to be zero which will lead to the maximum possible short circuit currents. The one-line diagram of the primary system to be used for the short circuit studies is shown in Figure 7.

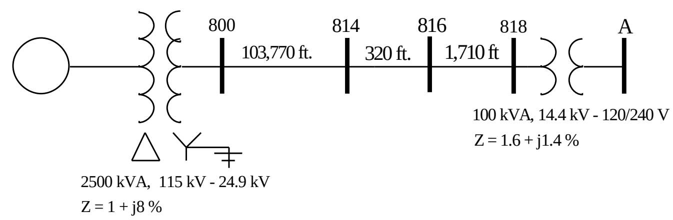

Figure 7 – Equivalent Primary System

In the IEEE 34 node test feeder the three-phase line between nodes 800 and 814 consist of 1/0 ACSR conductors for the phase and neutral using the ID 500 spacing. The line between nodes 814 and 816 consist of #2 6/1 ACSR conductors for the phase and neutral using the ID 500 spacing. The single-phase line between nodes 816 and 818 uses #4 6/1 ACSR conductors with the spacing ID of 510.

The ABC phase impedance matrix for the 2500 kVA substation transformer referred to the 24.9 kV side is:

$$ZT = \begin{pmatrix} 2.48+19.8403j & 0 & 0 \\ 0 & 2.48+19.8403j & 0 \\ 0 & 0 & 2.48+19.8403j \end{pmatrix} \quad \Omega \tag{29}$$

The ABC phase impedance matrix for the line between nodes 800 and 814 is:

$$Z_{300} = \begin{pmatrix} 27.1267+25.8291j & 4.9455+10.9757j & 5.02+9.4687j \\ 4.9455+10.9757j & 26.7963+26.2985j & 4.8553+8.6492j \\ 5.02+9.4687j & 4.8553+8.6492j & 26.9395+26.0945j \end{pmatrix} \quad \Omega \tag{30}$$

The ABC phase impedance matrix for the line between nodes 814 and 816 is:

$$Z_{301} = \begin{pmatrix} 0.117+0.0855j & 0.0141+0.039j & 0.0143+0.0345j \\ 0.0141+0.039j & 0.1161+0.0866j & 0.0139+0.0317j \\ 0.0143+0.0345j & 0.0139+0.0317j & 0.1165+0.0861j \end{pmatrix} \Omega \tag{31}$$

The ABC phase impedance matrix between nodes 816 and 818 is:

$$Z_{302} = \begin{pmatrix} 0.9114+0.4735j & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Omega \tag{32}$$

The total ABC phase impedance matrix from the substation to the center tapped transformer is the sum of the four matrices (Eqs. 29, 30, 31 and 31).

$$ZT_{ABC} = \begin{pmatrix} 30.6352+46.2285j & 4.9596+11.0147j & 5.0343+9.5032j \\ 4.9596+11.0147j & 29.3925+46.2253j & 4.8691+8.6809j \\ 5.0343+9.5032j & 4.8691+8.6809j & 29.536+46.0209j \end{pmatrix} \Omega \tag{33}$$

Since the center-tapped transformer is connected from phase A to ground, only the 1,1 term of ZT_{ABC} is used. With reference to Figures 5 and 6 the short circuit currents are shown in Table 4:

Table 4

Short Circuit Currents

	Line-to-Ground		Line-to-Line	
Node	I_A (amps)	I_1 (amps)	I_A (amps)	I_1 (amps)
818	261.5			
A	126.8	15,190	145.7	8790
B & E	21.3	2,550	58.5	3,516
C & F	11.6	1,387	35.7	2,141
D & G	7.9	950	17.6	1,053

Note in Table 4 the extremely high current when a line-to-ground short circuit occurs on the secondary of the transformer. Note also that the short circuit currents for faults at the customer's meter were also calculated.

IX. Conclusions

A model for a center tapped single-phase transformer has been developed along with the model for a three wire triplex cable. The model for the transformer for power-flow studies was in the form of matrices as was the model for the triplex cable. Data from a paper presented at the 2008 IEEE Transmission and Distribution Conference [5] were used to determine the input voltage to a center-tapped transformer. This voltage was then used as the input voltage for a power-flow study of a 28 customer secondary. The results of this study showed that several customers would not be receiving a voltage within the ANSI standards. The major purpose for modeling the transformer and the secondary system was to demonstrate that it is possible to accurately determine the voltage at a customer's meter.

The matrix model of the transformer does not work for short circuit studies. Instead a circuit analysis method was developed to compute the short circuit currents everywhere on the secondary system.

The models developed in this paper can and should be used by distribution engineers to accurately determine the operating conditions of a secondary system for power-flow and short-circuit studies.

X. References

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XI. Biography

W. H. Kersting (SM'64, F'89, Life Fellow 2003) was born in Santa Fe, NM. He received the BSEE degree from New Mexico State University, Las Cruces, and the MSEE degree from Illinois Institute of Technology. He joined the faculty at New Mexico State University in 1962 and served as Professor of Electrical Engineering and Director of the Electric Utility Management Program until his retirement in 2002. He is currently a consultant for Milsoft Utility Solutions. He is also a partner in WH Power Consultants, Las Cruces, NM.